

MATHEMATICS

Textbook for Class XI



11076



राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

ISBN 81-7450-486-9

First Edition

February 2006 Phalguna 1927

Reprinted

October 2006 Kartika 1928
November 2007 Kartika 1929
December 2008 Pausa 1930
December 2009 Agrahayana 1931
January 2011 Pausa 1932
February 2012 Magha 1933
December 2012 Pausa 1934
November 2013 Kartika 1935
December 2014 Pausa 1936
May 2016 Vaishakha 1938
December 2016 Pausa 1938
December 2017 Agrahayana 1939
January 2019 Pausa 1940

PD 450T BS

© National Council of Educational
Research and Training, 2006

₹ 210.00

Printed on 80 GSM paper with NCERT
watermark

Published at the Publication Division
by the Secretary, National Council of
Educational Research and Training,
Sri Aurobindo Marg, New Delhi 110 016
and printed at Sam Industrial
Enterprises Ltd., A 17-18, Sector – 60,
Noida - 201 301 (U.P.)

ALL RIGHTS RESERVED

- No part of this publication may be reproduced, stored in a retrieval system or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise without the prior permission of the publisher.
- This book is sold subject to the condition that it shall not, by way of trade, be lent, re-sold, hired out or otherwise disposed of without the publisher's consent, in any form of binding or cover other than that in which it is published.
- The correct price of this publication is the price printed on this page. Any revised price indicated by a rubber stamp or by a sticker or by any other means is incorrect and should be unacceptable.

**OFFICES OF THE PUBLICATION
DIVISION, NCERT**

NCERT Campus
Sri Aurobindo Marg
New Delhi 110 016 Phone : 011-26562708

108, 100 Feet Road
Hosdakere Halli Extension
Banashankari III Stage
Bengaluru 560 085 Phone : 080-26725740

Navjivan Trust Building
P.O. Navjivan
Ahmedabad 380 014 Phone : 079-27541446

CWC Campus
Opp. Dhankal Bus Stop
Panihati
Kolkata 700 114 Phone : 033-25530454

CWC Complex
Maligaon
Guwahati 781 021 Phone : 0361-2674869

Publication Team

Head, Publication Division : M. Siraj Anwar
Chief Editor : Shveta Uppal
Chief Business Manager : Gautam Ganguly
Chief Production Officer : Arun Chitkara
Editor : Bijnan Sutar
Production Officer : Abdul Naim

Cover and Layout
Arvinder Chawla

Foreword

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the Textbook Development Committee responsible for this

book. We wish to thank the Chairperson of the advisory group in Science and Mathematics, Professor J.V. Narlikar and the Chief Advisor for this book Professor P.K. Jain for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time and contribution. As an organisation committed to the systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

New Delhi
20 December 2005

Director
National Council of Educational
Research and Training

Textbook Development Committee

CHAIRPERSON, ADVISORY GROUP IN SCIENCE AND MATHEMATICS

J.V. Narlikar, *Emeritus Professor*, Chairman, Advisory Committee Inter University Centre for Astronomy & Astrophysics (IUCCA), Ganeshkhind, Pune University, Pune

CHIEF ADVISOR

P.K. Jain, *Professor*, Department of Mathematics, University of Delhi, Delhi

CHIEF COORDINATOR

Hukum Singh, *Professor*, DESM, NCERT, New Delhi

MEMBERS

A.K. Rajput, *Associate Professor*, RIE Bhopal, M.P.

A.K. Wazalwar, *Associate Professor*, DESM NCERT, New Delhi

B.S.P. Raju, *Professor*, RIE Mysore, Karnataka

C.R. Pradeep, *Assistant Professor*, Department of Mathematics, Indian Institute of Science, Bangalore, Karnataka.

Pradeep Hore, *Sr. Maths Master*, Sarla Birla Academy Bangalore, Karnataka.

S.B. Tripathy, *Lecturer*, Rajkiya Pratibha Vikas Vidyalaya, Surajmal Vihar, Delhi.

S.K.S. Gautam, *Professor*, DESM, NCERT, New Delhi

Sanjay Kumar Sinha, *P.G.T.*, Sanskriti School Chanakyapuri, New Delhi.

Sanjay Mudgal, *Lecturer*, CIET, New Delhi

Sneha Titus, *Maths Teacher*, Aditi Mallya School Yelaharika, Bangalore, Karnataka

Sujatha Verma, *Reader* in Mathematics, IGNOU, New Delhi.

Uday Singh, *Lecturer*, DESM, NCERT, New Delhi.

MEMBER-COORDINATOR

V.P. Singh, *Associate Professor*, DESM, NCERT, New Delhi

Acknowledgements

The Council gratefully acknowledges the valuable contributions of the following participants of the Textbook Review Workshop: P. Bhaskar Kumar, *P.G.T.*, Jawahar Navodaya Vidyalaya, Ananthpur, (A.P.); Vinayak Bujade, *Lecturer*, Vidarbha Buniyadi Junior College, Sakkardara Chowk Nagpur, Maharashtra; Vandita Kalra, *Lecturer*, Sarvodaya Kanya Vidyalaya Vikashpuri District Centre, New Delhi; P.L. Sachdeva, Deptt. of Mathematics, Indian Institute of Science, Bangalore, Karnataka; P.K. Tiwari, *Assistant Commissioner (Retd.)*, Kendriya Vidyalaya Sangathan; Jagdish Saran, Department of Statistics, University of Delhi; Quddus Khan, *Lecturer*, Shibli National P.G. College Azamgarh (U.P.); Sumat Kumar Jain, *Lecturer*, K.L. Jain Inter College Sasni Hathras (U.P.); R.P. Gihare, *Lecturer* (BRC), Janpad Shiksha Kendra Chicholi Distt. Betul (M.P.); Sangeeta Arora, *P.G.T.*, A.P.J. School Saket, New Delhi; P.N. Malhotra, *ADE* (Sc.), Directorate of Education, Delhi; D.R. Sharma, *P.G.T.*, J.N.V. Mungespur, Delhi; Saroj, *P.G.T.* Government Girls Sr. Secondary School, No. 1, Roop Nagar, Delhi; Manoj Kumar Thakur, *P.G.T.*, D.A.V. Public School, Rajender Nagar, Sahibabad, Ghaziabad (U.P.) and R.P. Maurya, *Reader*, DESM, NCERT, New Delhi.

Acknowledgements are due to Professor M. Chandra, *Head*, Department of Education in Science and Mathematics for her support.

The Council acknowledges the efforts of the Computer Incharge, Deepak Kapoor; Rakesh Kumar, Kamlesh Rao and Sajjad Haider Ansari, D.T.P. Operators; Kushal Pal Singh Yadav, Copy Editor and Proof Readers, Mukhtar Hussain and Kanwar Singh.

The contribution of APC–Office, administration of DESM and Publication Department is also duly acknowledged.

Contents

<i>Foreword</i>	<i>iii</i>
1. Sets	1
1.1 Introduction	1
1.2 Sets and their Representations	1
1.3 The Empty Set	5
1.4 Finite and Infinite Sets	6
1.5 Equal Sets	7
1.6 Subsets	9
1.7 Power Set	12
1.8 Universal Set	12
1.9 Venn Diagrams	13
1.10 Operations on Sets	14
1.11 Complement of a Set	18
1.12 Practical Problems on Union and Intersection of Two Sets	21
2. Relations and Functions	30
2.1 Introduction	30
2.2 Cartesian Product of Sets	30
2.3 Relations	34
2.4 Functions	36
3. Trigonometric Functions	49
3.1 Introduction	49
3.2 Angles	49
3.3 Trigonometric Functions	55
3.4 Trigonometric Functions of Sum and Difference of Two Angles	63
3.5 Trigonometric Equations	74
4. Principle of Mathematical Induction	86
4.1 Introduction	86
4.2 Motivation	87
4.3 The Principle of Mathematical Induction	88

5.	Complex Numbers and Quadratic Equations	97
5.1	Introduction	97
5.2	Complex Numbers	97
5.3	Algebra of Complex Numbers	98
5.4	The Modulus and the Conjugate of a Complex Number	102
5.5	Argand Plane and Polar Representation	104
5.6	Quadratic Equations	108
6.	Linear Inequalities	116
6.1	Introduction	116
6.2	Inequalities	116
6.3	Algebraic Solutions of Linear Inequalities in One Variable and their Graphical Representation	118
6.4	Graphical Solution of Linear Inequalities in Two Variables	123
6.5	Solution of System of Linear Inequalities in Two Variables	127
7.	Permutations and Combinations	134
7.1	Introduction	134
7.2	Fundamental Principle of Counting	134
7.3	Permutations	138
7.4	Combinations	148
8.	Binomial Theorem	160
8.1	Introduction	160
8.2	Binomial Theorem for Positive Integral Indices	160
8.3	General and Middle Terms	167
9.	Sequences and Series	177
9.1	Introduction	177
9.2	Sequences	177
9.3	Series	179
9.4	Arithmetic Progression (A.P.)	181
9.5	Geometric Progression (G.P.)	186
9.6	Relationship Between A.M. and G.M.	191
9.7	Sum to n terms of Special Series	194
10.	Straight Lines	203
10.1	Introduction	203
10.2	Slope of a Line	204
10.3	Various Forms of the Equation of a Line	212
10.4	General Equation of a Line	220
10.5	Distance of a Point From a Line	225

11. Conic Sections	236
11.1 Introduction	236
11.2 Sections of a Cone	236
11.3 Circle	239
11.4 Parabola	242
11.5 Ellipse	247
11.6 Hyperbola	255
12. Introduction to Three Dimensional Geometry	268
12.1 Introduction	268
12.2 Coordinate Axes and Coordinate Planes in Three Dimensional Space	269
12.3 Coordinates of a Point in Space	269
12.4 Distance between Two Points	271
12.5 Section Formula	273
13. Limits and Derivatives	281
13.1 Introduction	281
13.2 Intuitive Idea of Derivatives	281
13.3 Limits	284
13.4 Limits of Trigonometric Functions	298
13.5 Derivatives	303
14. Mathematical Reasoning	321
14.1 Introduction	321
14.2 Statements	321
14.3 New Statements from Old	324
14.4 Special Words/Phrases	329
14.5 Implications	335
14.6 Validating Statements	339
15. Statistics	347
15.1 Introduction	347
15.2 Measures of Dispersion	349
15.3 Range	349
15.4 Mean Deviation	349
15.5 Variance and Standard Deviation	361
15.6 Analysis of Frequency Distributions	372

16. Probability	383
16.1 Introduction	383
16.2 Random Experiments	384
16.3 Event	387
16.4 Axiomatic Approach to Probability	394
Appendix 1: Infinite Series	412
A.1.1 Introduction	412
A.1.2 Binomial Theorem for any Index	412
A.1.3 Infinite Geometric Series	414
A.1.4 Exponential Series	416
A.1.5 Logarithmic Series	419
Appendix 2: Mathematical Modelling	421
A.2.1 Introduction	421
A.2.2 Preliminaries	421
A.2.3 What is Mathematical Modelling	425
Answers	433
Supplementary Material	466